

22. MATHEMATICS

UNIT – I

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limit supremum, limit infimum. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence.

Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral.

Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation.

Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.

Algebra of matrices, rank and determinant of matrices, linear equations. Eigen values and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

UNIT – II

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, Calculus of residues. Conformal mappings, Mobius transformations.

Algebra: Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in \mathbb{Z} , congruences, Chinese Remainder Theorem, Euler's ϕ -function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.

UNIT – III

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis : Numerical solutions of algebraic equations: Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations: Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Classical Mechanics: Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

Unit-IV

Mathematical Statistics:

Collection, tabulation and representation of data, measures of central tendency and dispersion, moments, skewness and kurtosis.

Sample space and events with classical, empirical and axiomatic definition of probability, independent events, conditional probability, Baye's theorem, random variables and distribution functions. Moment generating function, Characteristic function. Probabilistic inequalities (Tchebychev, Holder and Jensen). Weak and strong laws of large numbers. Central limit theorem (i.i.d. case).

Discrete and continuous univariate distributions namely; Binomial, Poisson and Normal.

Bivariate data, scatter diagram, Simple Correlation and Rank correlation, Regression lines, Multiple and partial correlations (three variables case only).

Concept of sampling and statistic, simple random sampling with and without replacement, Stratified sampling, Probability proportional to size sampling.

Statistical Inference: Statistic, estimates and estimator. Requirements of a good estimator-unbiasedness, consistency, efficiency and sufficiency. Methods of moments and likelihood. Simple and composite hypotheses, null and and alternation hypotheses. critical region, two types of errors, level of significance and power of a test. Neyman Pearson's lemma and its application. Tests based on t, Z, F and χ^2 . Analysis of variance one way and two way classification.

Operations Research:

Linear Programming Problem, Simplex methods, Dual of an LPP, Dual Simplex Method. Elementary Queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1 and inventory models.

Formulation of Transportation Problems, Finding initial basic feasible solution, Test of optimality, MODI method, Degeneracy, Stepping Stone method, Solutions of Assignment problems, Hungarian Method.